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THE

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THE

MATHEMATICAL MONTHLY.

Vol. II. . . . MAY, 1860. . . . No. VIII.

PRIZE PROBLEMS FOR STUDENTS.

I. If x is a whole number, prove that $x^3 - x$ is always divisible by 6.

II. Describe a series of circles whose areas shall be one half, one fourth, one eighth, &c. that of a given circle. — Communicated by George Stuart, Haverford College, Pa.

III. If R and r be the radii of the circles circumscribing and inscribing any triangle, and D the distance between their centres, then $D^2 = R^2 - 2Rr$. Required, a geometrical demonstration. — Communicated by George Eastwood, Saxonville, Mass.

IV. If a circle cut a conic section in four points, A, B, C, D, and a second circle cut the same conic section in A, B, E, F, then will CD and EF be parallel. — Communicated by Asher B. Evans, Madison University, Hamilton, N. Y.

V. In any conic section let u denote the length of the perpendicular dropped from any point in the curve upon the directrix, r the distance of this point from the nearest focus, a and b the semi-axes; prove that

$$\frac{u^2 - r^2}{u^2} = \frac{b^2}{a^2}.$$

— Communicated by David Trowbridge, Perry City, N. Y.

Solutions of these problems must be received by July 1st, 1860.

REPORT OF THE JUDGES UPON THE SOLUTIONS OF THE PRIZE PROBLEMS IN No. V., Vol. II.

The first Prize is awarded to David Trowbridge, Perry City, Schuyler Co., N. Y.

The second Prize is awarded to George B. Hicks, Cleveland, Ohio.

The third Prize is awarded to Miss Harriet S. Hazeltine, Worcester, Mass.

Prize Solution of Problem III.

By G. B. Hicks and David Trowbridge.

III. Find the n quantities $x_1, x_2, \ldots\ldots x_n$, from the n equations,

$$(1) \qquad x_1 + x_2 + x_3 \ldots\ldots + x_n = A,$$

$$(2) \qquad a_1 x_1 + a_2 x_2 + a_3 x_3 \ldots\ldots + a_n x_n = 0,$$

$$(3) \qquad a_1^2 x_1 + a_2^2 x_2 + a_3^2 x_3 \ldots\ldots + a_n^2 x_n = 0,$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$(n) \qquad a_1^{n-1} x_1 + a_2^{n-1} x_2 + a_3^{n-1} x_3 \ldots\ldots + a_n^{n-1} x_n = 0,$$

and obtain symmetrical expressions for x_1, x_2, &c.

If we multiply each of the equations (1), (2), ... $(n-1)$, by a_1, and subtract (2) from the first product, (3) from the second, and so on, we obtain the $n-1$ equations,

$$(1)' \qquad (a_1 - a_2)\, x_2 + (a_1 - a_3)\, x_3 \ldots\ldots + (a_1 - a_n)\, x_n = A\, a_1,$$

$$(2)' \qquad a_2 (a_1 - a_2)\, x_2 + a_3 (a_1 - a_3)\, x_3 \ldots\ldots + a_n (a_1 - a_n)\, x_n = 0,$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$(n-1)' \quad a_2^{n-2}(a_1 - a_2) x_2 + a_3^{n-2}(a_1 - a_3) x_3 \ldots\ldots + a_n^{n-2}(a_1 - a_n) x_n = 0,$$

from which x_1 is eliminated.

If now we multiply each of these equations but the last by a_2, and from the first product subtract $(2)'$, from the second $(3)'$, and so on, we shall get $n-2$ equations from which x_2 will be eliminated. Operating on these $n-2$ equations, and then on those which result in a similar manner, and so on, we shall finally obtain

$$(a_1 - a_n)\,(a_2 - a_n) \ldots\ldots (a_{n-1} - a_n)\, x_n = A\, a_1 a_2 \ldots\ldots a_{n-1}.$$

$$\therefore x_n = \frac{A a_1 a_2 \ldots\ldots a_{n-1}}{(a_1 - a_n)(a_2 - a_n) \ldots\ldots (a_{n-1} - a_n)}.$$

Since a precisely similar process will give any other value of x, it is plain that all the values may be obtained from this one by permutation. The solutions by ASHER B. EVANS and T. E. TOWER are essentially the same as the one here given.

PRIZE SOLUTION OF PROBLEM IV.

By G. B. HICKS and GUSTAVUS FRANKENSTEIN.

IV. Prove that $\sin^n(\theta - \varphi)\sin\varphi$ is a maximum when

$$\sin(\theta - 2\varphi) = \frac{n-1}{n+1}\sin\theta;$$

θ being a given constant, and φ the variable.

If a function is a maximum or a minimum its logarithm will be also; and therefore we may put

$$u = n \log\sin(\theta - \varphi) + \log\sin\varphi.$$

Taking the derivative

$$\frac{du}{d\varphi} = -\frac{n\cos(\theta - \varphi)}{\sin(\theta - \varphi)} + \frac{\cos\varphi}{\sin\varphi} = 0,$$

$$\therefore \sin(\theta - \varphi)\cos\varphi = n\cos(\theta - \varphi)\sin\varphi. \tag{1}$$

Now adding $\cos(\theta - \varphi)\sin\varphi$ to both sides of (1), and then subtracting it from both sides, we get

$$\sin\theta = (n+1)\cos(\theta - \varphi)\sin\varphi,$$

and
$$\sin(\theta - 2\varphi) = (n-1)\cos(\theta - \varphi)\sin\varphi.$$

$$\therefore \frac{\sin\theta}{n+1} = \frac{\sin(\theta - 2\varphi)}{n-1}; \quad \text{or } \sin(\theta - 2\varphi) = \frac{n-1}{n+1}\sin\theta.$$

Taking the derivative of $\frac{du}{d\varphi}$ gives

$$\frac{d^2u}{d\varphi^2} = -(n\,\mathrm{cosec}^2(\theta - \varphi) + \mathrm{cosec}^2\varphi),$$

which is essentially negative so long as n is positive, and therefore the given function is a maximum.

Prize Solution of Problem V.

By M. K. Bosworth, Marietta College, Ohio.

V. The notation of Problem V. in the November No. being retained, prove that in the plane triangle

(1) $$\frac{1}{\rho'}+\frac{1}{\rho''}+\frac{1}{\rho'''}=\frac{1}{\rho};$$

and in the spherical triangle,

(2) $$\frac{1}{\tan\rho'}+\frac{1}{\tan\rho''}+\frac{1}{\tan\rho'''}=\frac{1}{\tan\rho}\,\frac{\cos(\rho+\delta)\cos(\rho-\delta)}{\cos^2 r\cos^2\rho};$$

(3) $$\frac{\cos\delta'}{\sin\rho'}+\frac{\cos\delta''}{\sin\rho''}+\frac{\cos\delta'''}{\sin\rho'''}=\frac{\cos\delta}{\sin\rho}.$$

In the plane triangle (Chauvenet's Trig. Eqs. 289 and 293)

$$\frac{1}{\varrho}=\frac{s}{k},\quad \frac{1}{\varrho'}=\frac{s-a}{k},\quad \frac{1}{\varrho''}=\frac{s-b}{k},\quad \frac{1}{\varrho'''}=\frac{s-c}{k},$$

and therefore (1) is true. In the spherical triangle

$$\frac{1}{\tan\varrho}=\frac{\sin s}{k},\ \frac{1}{\tan\varrho'}=\frac{\sin(s-a)}{k},\ \frac{1}{\tan\varrho''}=\frac{\sin(s-b)}{k},\ \frac{1}{\tan\varrho'''}=\frac{\sin(s-c)}{k};$$

$$\therefore\quad \frac{1}{\tan\varrho'}+\frac{1}{\tan\varrho''}+\frac{1}{\tan\varrho'''}=\frac{\sin(s-a)+\sin(s-b)+\sin(s-c)}{k}.$$

But from *Math. Monthly*, Vol. II., p. 229, and $\tan\varrho$, we have

$$\frac{1}{\tan\varrho}\,\frac{\cos(\varrho+\delta)\cos(\varrho-\delta)}{\cos^2 r\cos^2\varrho}=\frac{\sin(s-a)+\sin(s-b)+\sin(s-c)}{k};$$

and therefore (2) is true.

Also, from *Math. Monthly*, Vol. II., p. 150; Chauvenet's Trig., p. 251, and Eqs. 319 and 323, we get

$$\frac{\cos\delta'}{\sin\varrho'}=\frac{-\sin a+\sin b+\sin c}{2k}\cos r,$$

$$\frac{\cos\delta''}{\sin\varrho''}=\frac{\sin a-\sin b+\sin c}{2k}\cos r,$$

$$\frac{\cos\delta'''}{\sin\varrho'''}=\frac{\sin a+\sin b-\sin c}{2k}\cos r.$$

$$\therefore\ \frac{\cos\delta'}{\sin\varrho'}+\frac{\cos\delta''}{\sin\varrho''}+\frac{\cos\delta'''}{\sin\varrho'''}=\frac{\sin a+\sin b+\sin c}{2k}\cos r=\frac{\cos\delta}{\sin\varrho}.$$

Simon Newcomb.
W. P. G. Bartlett.
Truman Henry Safford.

NOTES AND QUERIES.

1. *Note on Co-Factors.* — If one factor of a product is known, the other may often be found by inspection. For example, 179 being one factor of 17363, the other factor must be 97, because 7 is the only digit that, when multiplied by 9, gives a product terminating in 3. If the product is an odd number, and not divisible by 5, the right-hand figure of the unknown factor may always be determined, and the left-hand figure may be ascertained as in ordinary division.

If the factor sought contains three figures, the middle figure may generally be found by *casting out nines.*

Let $9n + a$, and $9n_1 + x + y$, be the two factors of any given number $9n_2 + b$. Multiplying the two factors together, we find that $a(x + y) = 9n_3 + b$. The values of a and b being known, the value of $x + y$ may be ascertained by substituting for n_3 the successive values 0, 1, 2, &c., until we obtain a value for $9n_3 + b$ that is divisible by a, and adding $9n_4$ to the quotient, (n_4 being equal to either 0, 1, or 2,) x may be considered as equivalent to the sum of the right-hand and left-hand digits of the unknown factor, and $y = \frac{9n_3 + b}{a} - x$, will then be the middle digit.

Let it now be required to determine by inspection the value of z, in the equation $9769z + 471 = 8333428$. Transposing, $9769z = 8332957$. Rejecting the 9's from 9769 we obtain $a = 4$; rejecting the 9's from 8332957 gives $b = 1$. The least value of n_3, that makes $9n_3 + b$ divisible by 4, is 3; $\therefore x + y = 9n_4 + 7$. The left-hand digit is evidently 8, and the right-hand digit 3;

$$\therefore x = 11; \qquad y = 5; \qquad z = 853.$$

If $a = 0$, y may have ten possible values (from 0 to 9 inclusive). If $a = 3$, or 6, y may have three possible values. For any other value of a there can be only one possible value of y.

Table I. will give the right-hand digit, except when $a = 0$, 3, or 6. The value of a is to be sought in either margin, b in the body of the table, on a line with a, and $x + y - 9n_4$ in the other margin on a line with b.

TABLE I.

	1	2	4	5	7	8
1	1	2	4	5	7	8
2	2	4	8	1	5	7
4	4	8	7	2	1	5
5	5	1	2	7	8	4
7	7	5	1	8	4	2
8	8	7	5	4	2	1

TABLE II.

1	2	4	5	7	8
1, 1	1, 2	1, 4	1, 5	1, 7	1, 8
2, 5	4, 5	2, 2	2, 7	2, 8	2, 4
4, 7	7, 8	5, 8	4, 8	4, 4	5, 7
8, 8		7, 7		5, 5	

Table II. gives all the possible forms of the two co-factors ($9n + a$, $9n_1 + a_1$) of any composite number $9n_2 + b$, except when $a = 0$, 3, or 6. The value of b is to be sought in the upper line, and the possible values of a, a_1, will be found in the column below.

Example. — 12697 ($= 9n_2 + 7$) must either be a prime number, or if it have factors, they must be of the form $(9n + 1)(9n_1 + 7)$, $(9n + 2)(9n_1 + 8)$, $(9n + 4)(9n_1 + 4)$, or $(9n + 5)(9n_1 + 5)$. — PLINY EARLE CHASE, Philadelphia.

2. *Proposition in the Theory of Numbers.* — Let any number be separated into periods of three figures each, counting from the right; then if the difference of the sums of the odd (1, 3, 5) and of the even (2, 4, 6) periods be zero, or be divisible by 7, 11, or 13, the number itself will be divisible by 7, 11, or 13.

If we divide the successive powers of 10 by 7, attending to the remainders only, which may be considered either positive or negative, we shall have

$$10 = 7n - 4, \quad 10^3 = 7n'' - 1, \quad 10^5 = 7n^{IV} + 5,$$
$$10^2 = 7n' - 5, \quad 10^4 = 7n''' + 4, \quad 10^6 = 7n^{V} + 1.$$

It is evident that if the division be further continued the remain-

ders will follow each other in the order of the series, —4, —5, —1, +4, +5, +1; and generally, the powers of 10 whose exponents are multiples of 3, and odd, will be of the form $7n - 1$, and those whose exponents are multiples of 3, and even, of the form $7n' + 1$. Now let there be any number $a\,b\,c\,d\,e\,f\,g\,h\,i\,k$, and let it be separated into periods of three figures. For the sake of convenience, let each period be denoted by a single letter; then

$$a,\ bcd,\ efg,\ hik = lmpq.$$

The number $lmpq$ may be put under the form

$$\begin{aligned} lmpq &= 10^9 l + 10^6 m + 10^3 p + q, \\ &= (7n - 1)\,l + (7n' + 1)\,m + (7n'' - 1)\,p + q, \\ &= 7(n + n' + n'') - l + m - p + q. \end{aligned}$$

Now the part $7(n + n' + n'')$ is divisible by 7, consequently, if the part $-l + m - p + q$ is divisible by 7 the number $lmpq$ is also. But this latter part is equal to the difference of the sums of the odd and even periods into which the given number was separated.

By the same method the proposition may be proved true for the numbers 11 and 13. — H. Willey, Esq., New Bedford, Mass.

3. *Note on the Ox Question.* — Would not the following come more properly under the definition of an *analytical* solution?

"If a oxen in m weeks eat b acres of grass, and c oxen in n weeks eat d acres, how many oxen will eat e acres in p weeks, the grass being supposed to grow uniformly?"

Let x = the required number, α the grass at first on an acre, and β the growth of one acre in one week; then

$$\begin{aligned} b(\alpha + m\beta) &= \text{grass on } b \text{ acres at end of } m \text{ weeks}, \\ d(\alpha + n\beta) &= \text{“ } d \text{ “ “ } n \text{ “}, \\ e(\alpha + p\beta) &= \text{“ } e \text{ “ “ } p \text{ “} \end{aligned}$$

The quantity of grass consumed will be proportional to *both the number of oxen and the time together;* hence

(1) $$b(\alpha + m\beta) : d(\alpha + n\beta) :: ma : nc,$$

(2) $$b(\alpha + m\beta) : e(\alpha + p\beta) :: ma : px.$$

(1) gives $\beta = \frac{(maa - ncb)\alpha}{mn(bc - ad)}$, (2) gives $x = \frac{mae(\alpha + p\beta)}{pb(\alpha + m\beta}$.

$$\therefore x = \left(\frac{m-p}{m-n}\right)\frac{nce}{pd} - \left(\frac{n-p}{m-n}\right)\frac{mae}{pb}.$$

Any one of the quantities may be assumed as unknown, and its value obtained from this last equation. — SAMUEL SCHOOLER, Principal of Edge Hill School, Guiney's P. O., Va.

4. *Right-Angled Triangles with commensurable Sides.* — Teachers frequently have occasion to represent the sides of right-angled triangles by exact numbers. Three, four, and five, or equimultiples of those numbers, are often employed, so often in fact that pupils sometimes suppose no others can be used. The following facts may be of service in giving variety to problems, or in illustrating algebraic formulas relating to the right-angled triangle.

There are sixteen dissimilar right-angled triangles with commensurable sides, including only those in which the hypothenuse contains the greatest common measure less than one hundred times. The relations of their sides may be represented by

$3^2 + 4^2 = 5^2$, $5^2 + 12^2 = 13^2$, $8^2 + 15^2 = 17^2$, $7^2 + 24^2 = 25^2$,
$20^2 + 21^2 = 29^2$, $12^2 + 35^2 = 37^2$, $9^2 + 40^2 = 41^2$, $28^2 + 45^2 = 53^2$,
$11^2 + 60^2 = 61^2$, $16^2 + 63^2 = 65^2$, $33^2 + 56^2 = 65^2$, $48^2 + 55^2 = 73^2$,
$13^2 + 84^2 = 85^2$, $36^2 + 77^2 = 85^2$, $39^2 + 80^2 = 89^2$, $65^2 + 72^2 = 97^2$.

The greatest common measure, here represented by unity, may be of any length, and hence there may be an infinite number of similar triangles represented by each of the above equations. If unity is supposed to represent an absolute length, as an inch, a foot, or a

yard, then the similar triangles will be represented by like parts and equimultiples of the numbers composing each equation, such as $(\frac{3}{2})^2 + 2^2 = (\frac{5}{2})^2$, and $6^2 + 8^2 = 10^2$. Rejecting all fractions, and all numbers above 100, the first of the above equations gives 20 similar triangles, the second 7, the third 5, the fourth 4, the fifth 3, and the sixth and seventh 2 each, making in all, 52 right-angled triangles, whose sides may be represented by integers not exceeding 100, when unity represents some fixed linear unit. Those represented by the fifth equation approach nearest to the isosceles right-angled triangle, and that represented by the thirteenth equation has the greatest difference between the sides including the right angle.

When the sides of a right-angled triangle are commensurable, the perpendicular let fall from the vertex of the right angle to the hypothenuse is commensurable with the sides, because the two partial triangles thus formed are similar to the original one. In other words, when the sides of a right-angled triangle can be exactly expressed by numbers, the perpendicular can also be exactly expressed, usually by the aid of fractions, as the *greatest* common measure of the sides does not measure the perpendicular. In such a triangle, the diameter of the inscribed circle is also commensurable with the sides, as it is equal to the sum of the other two sides minus the hypothenuse. The perpendicular is equal to the product of the other two sides divided by the hypothenuse. — Prof. D. W. Hoyt, Fairfax, Vt.

5. The method of developing the expression $A = \frac{15\,(E - \sin E)}{9\,E + \sin E}$, employed by Prof. Cotter, in the Mathematical Monthly for March, p. 185, is not the most natural, and does not exhibit the law of progression of the coefficients of the resulting series otherwise than by inspection of the result when obtained. By pursuing the following more direct process, the law of progression is evident at each step.

We have to develop A in terms of T, having $T = \tan^2 \frac{1}{2} E$. We take

$$E = 2 \tan^{-1} T^{\frac{1}{2}} = 2 T^{\frac{1}{2}} (1 - \tfrac{1}{3} T + \tfrac{1}{5} T^2 - \tfrac{1}{7} T^3 + \tfrac{1}{9} T^4 - \&c.)$$

$$\sin E = \frac{2 T^{\frac{1}{2}}}{1 + T} = 2 T^{\frac{1}{2}} (1 + T)^{-1}$$

$$= 2 T^{\frac{1}{2}} (1 - T + T^2 - T^3 + T^4 - \&c.);$$

whence

$$15 (E - \sin E) = 30 T^{\frac{1}{2}} (\tfrac{2}{3} T - \tfrac{4}{5} T^2 + \tfrac{6}{7} T^3 - \tfrac{8}{9} T^4 + \&c.)$$

$$= 20 T^{\frac{1}{2}} (T - \tfrac{6}{5} T^2 + \tfrac{9}{7} T^3 - \tfrac{12}{9} T^4 + \&c.)$$

$$9 E + \sin E = 2 T^{\frac{1}{2}} (10 - \tfrac{12}{3} T + \tfrac{14}{5} T^2 - \tfrac{16}{7} T^3 + \&c.)$$

$$= 20 T^{\frac{1}{2}} (1 - \tfrac{6}{15} T + \tfrac{7}{25} T^2 - \tfrac{8}{35} T^3 + \&c.)$$

the substitution of which gives the required development,

$$A = \frac{T - \frac{6}{5} T^2 + \frac{9}{7} T^3 - \frac{12}{9} T^4 + \&c.}{1 - \frac{6}{15} T + \frac{7}{25} T^2 - \frac{8}{35} T^3 + \&c.}.$$

— Prof. W. Chauvenet, Washington University, St. Louis, Mo.

6. *Note on the Theory of Perspective.* — The theory of perspective may often be applied to the discovery and demonstration of geometrical truths. I propose to give a few examples of this application. Through the points A, B, C, in the same straight line, let

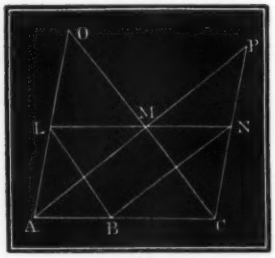


Fig. 1.

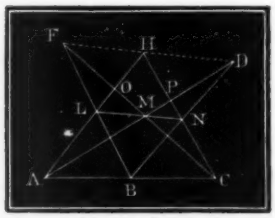


Fig. 2.

three sets of parallels be drawn, as in Fig. 1. The points L, M, N are in the same straight line, for

$$AL : LO = AB : BC = PN : NC.$$

Therefore the line LN passes through the intersection of OC and

AP. Fig. 2 is the projection of Fig. 1, FD being the horizon, if the parallel lines are horizontal. The points L, M, N are still in the same straight line; whence the following theorem of APOLLONIUS.

"If any quadrilateral be divided into two other quadrilaterals by a straight line, the intersections of the diagonals of the three quadrilaterals are on the same straight line."

As a second example, let us take a circle. Draw any two diameters, and at their extremities draw four tangents. If now we connect the intersections of the diameters and tangents by lines, and also draw tangents at the points where these lines cut the circle, a symmetrical figure will be formed. If we take the perspective of this figure, we may assume any ellipse for the projection of the circle, and any point within it for the projection of the centre. Any line drawn through this point may be taken as the projection of one of the diameters; the projection of the other is then determined by the angle which the original diameters make with each other. The original tangents are evidently tangents to the ellipse in the projection of the figure. We have then this general theorem.

I. Any two lines which are parallel in the original figure meet on the horizon in the perspective representation.

II. If any line is bisected in the original figure, this line prolonged to the horizon is divided harmonically.

Problem V. in the Math. Monthly, Vol. I. No. I., is included in the first part of this theorem. — G. B. VOSE, Assistant U. S. Coast Survey, Washington, D. C.

7. *Note on the "Rule of False."* — This rule admits of a very simple explanation, which has probably occurred to most persons who are in the habit of using the rule.

The explanation suggests the origin of the rule, and cannot be new; still I do not remember having seen it.

In any equation of one unknown quantity, such as $f(x) = 0$, let

us for a moment suppose x to be the variable abscissa of a curve, while y, the variable value of $f(x)$, is the ordinate. The equation becomes

$$f(x) = y.$$

We wish to find the point where the curve intersects the axis of x; that is, a value of x which shall make $y = 0$.

Find by trial two values of x, which we may designate by x_1, and x_2, which shall give two values of y, y_1 and y_2, nearly equal to zero. *The rule of false, or rule of double position, is based on the supposition that the curve near the axis of* x *is a right line.*

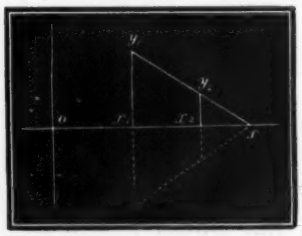


Fig. 1.

$$o\,x_1 = x_1$$
$$o\,x_2 = x_2$$
$$o\,x = x$$
$$x_1\,y_1 = y_1$$
$$x_2\,y_2 = y_2$$

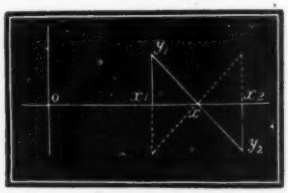


Fig 2.

If y_1 and y_2 are both positive or both negative, Fig. 1 gives

$$y_1 : y_2 :: (x - x_1) : (x - x_2); \qquad \text{or } x = \frac{y_1 x_2 - y_2 x_1}{y_1 - y_2}.$$

If y_1 be positive and y_2 negative, or y_2 positive and y_1 negative, Fig. 2 gives

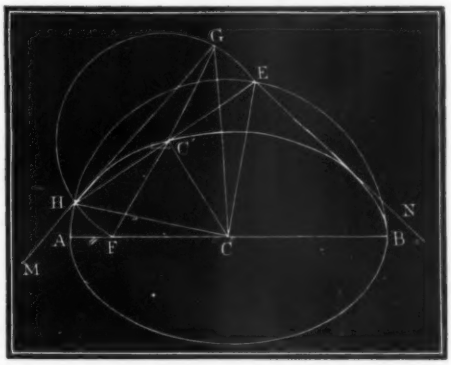
$$y_1 : y_2 :: (x - x_1) : (x_2 - x)\cdot \qquad \text{or } x = \frac{y_1 x_2 + y_2 x_1}{y_1 + y_2}.$$

These two formulæ, enunciated in words, make the "rule of false." It will be seen that the errors, so called, are true ordinates of the curve, and that the final result will be more or less accurate, according to the nature of the curve and the proximity of y_1 and y_2 to zero.

The rule as announced in arithmetics does not apply to the case in which the two "positions," x_1 and x_2, are unlike.

If we put the known term in the second member $f'(x) = p$, the solution will give the point or points where the curve intersects a line parallel to the axis of x at the distance p above it. — Capt. D. P. Woodbury, U. S. A.

9. *Solution of Prize Problem III. in Vol. I. No. V. — Problem.* "If two sides of a movable right angle are always tangent to a given ellipse, its summit will describe a circle concentric with the ellipse, the radius of which is equal to the chord joining the extremities of the major and minor axes."



Solution. Let F be the foci of any ellipse, and let GM and GN be the sides of a right angle tangent to it. Join GF, and on it as a diameter describe a segment of a circle, cutting the circle described on AB, the major axis, in E and H. It is evident that GM will pass through H, and GN through E. (See Coffin's Conic Sections, Prop. XV.) And since MGN is a right angle, HE is a diameter to the circle on GF, and hence it passes through C', its centre. From C, the centre of the ellipse, draw CH, CC', CG and CE. Represent the semi-axes as usual by A and B. Then (Legendre, Prop. XIV. B. IV.)

$$2HC'^2 + 2CC'^2 = CH^2 + CE^2 = 2A^2.$$

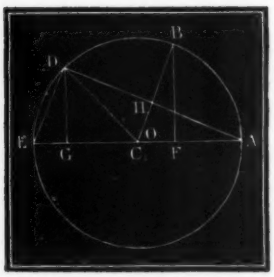
Also, $2FC'^2 + 2CC'^2 = FC'^2 + CG^2 = A^2 - B^2 + CG^2$

Whence, since $HC' = FC'$, $2A^2 = A^2 - B^2 + CG^2$.

Therefore $CG = \sqrt{(A^2 + B^2)}$, which was to be proved.

— M. C. Stevens, Prof. of Mathematics, Haverford College, Pa.

10. *Graphic Demonstration of the Formula* $2 \sin \varphi \cos \varphi = \sin 2\varphi$. — Let $BCA = \varphi =$ any angle; $ACD = 2\varphi$; $R =$ radius of the circle ABE.



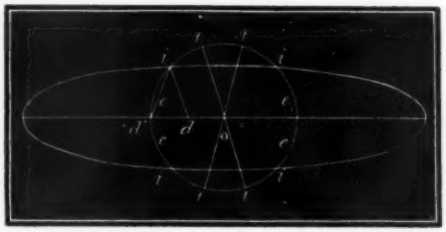
Drop a perpendicular from B to CA, BC is perpendicular to HA; therefore the right-angled triangles CBF and CAH are similar. They are equal, for the hypothenuses CB and CA are radii. They are also equal to DHC. It follows from this that the triangle $DCA = 2\,CBF = BF \times FC$.

From D drop a perpendicular to EC; the area of the triangle $ECD = \frac{1}{2}\,DG \times EC = \frac{1}{2}\,DG \times R$. The area of the triangle $EAD = \frac{1}{2}DG \times EA = \frac{1}{2}DG \times 2R = DG \times R$. But $DEA - DEC = BF \times FC = DCA = DG \times R - \frac{1}{2}DG \times R = \frac{1}{2}\,DG \times R$, therefore $DEC = DCA$. But $BF = R \sin \varphi$; $CF = R \cos \varphi$; $DG = R \sin 2\varphi$. We have just seen that twice the triangle $BCF =$ triangle DEC; i. e. $BF \times FC = \frac{1}{2}\,DG \times R$, therefore $R \sin \varphi \times R \cos \varphi = R \times \frac{1}{2}\,R \sin 2\varphi$; hence,

$$2 \sin \varphi \cos \varphi = \sin 2\varphi.$$

— E. Harrison, St. Louis, Mo.

11. *Note on the Trisection of an Arc.* — An ellipse and concentric circle are so drawn that $B < R$, and $A = B + 2R$. Required a geometrical trisection of each of the arcs intercepted on the circumference of the circle by the ellipse.



Solution. — From either of the four points of intersection, i, with a radius $= B$, cut A at d, d'. Through the centre, o, draw ot parallel to di, or $(d'i)$. Then

the exterior arc will be trisected at t. From t, with radius R, cut the circumference of the circle (on the same side of the ellipse's minor axis) at e. Then the interior arc will be trisected at e. The arcs may also be trisected by means of a radius $= A$.

Proof. — Extend id to t. Angle itt and tot are equal, and arc itt = twice tt. $\therefore itti$ is trisected at t and $180° - itti$ is trisected by ie, which $= 60° - it$.

NOTE. — SALMON'S "Treatise on Conic Sections" gives the problem of the trisection of circular arcs, the point of trisection being determined as the intersection of the given arc with a given hyperbola. I have never met with a solution of the problem by means of the ellipse. — PLINY EARLE CHASE, Philadelphia, Pa.

ON SPHERICAL ANALYSIS.

By GEORGE EASTWOOD, Saxonville, Mass.

[Continued from Page 190.]

PROPOSITION IV.

To find the equation of a great circle of the sphere in terms of the co-ordinates of its pole. — Sometimes it is more convenient to use the equation of a great circle expressed in terms of the co-ordinates of its pole, than the one expressed in terms of the intercepts of the axes. Suppose, therefore, that AB, in the annexed diagram, is the proposed great circle; that x_1, $\bar{y}_1$, are the geographical co-ordinates of its pole or *centre*, and that x, $\bar{y}$, are the geographical co-ordinates of any point P in its circumference. By inspection of the figure, $\frac{1}{2}\pi - \bar{y}_1$, $\frac{1}{2}\pi - \bar{y}$, and $\frac{1}{2}\pi$, are the sides of a spherical triangle whose vertex is at Y, and whose vertical angle is $x_1 - x$. Hence, by spherics, we have

$$\cos \tfrac{1}{2}\pi = \sin \bar{y} \sin \bar{y}_1 + \cos \bar{y} \cos \bar{y}_1 \cos (x_1 - x),$$

and by division and expansion, we have

$$\tan \bar{y} \tan \bar{y}_1 = -(\cos x \cos x_1 + \sin x \sin x_1).$$

But, when the axes are rectangular, we have $\tan \bar{y} = y \cos x$, $\tan \bar{y}_1 = y_1 \cos x_1$; the substitution of which in $\tan \bar{y} \tan \bar{y}_1$, gives, after dividing out $\cos x \cos x_1$, $y y_1 = -1 - x x_1$,

or
$$-\frac{y}{\frac{1}{y_1}} - \frac{x}{\frac{1}{x_1}} = 1.$$

Comparing this equation with $\frac{y}{\beta} + \frac{x}{\alpha} = 1$, we see that $\beta = -\frac{1}{y_1}$, and $\alpha = -\frac{1}{x_1}$.

Hence the equation of a great circle expressed in tangent-functions of the co-ordinates of its pole is

(22)
$$y y_1 + x x_1 + 1 = 0.$$

NOTES ON PROBABILITIES.

By Simon Newcomb, Nautical Almanac Office, Cambridge, Mass.

[Continued from Page 140.]

22. The *a priori* probability of an event is its probability deduced solely from circumstances which preceded or might have preceded it. In the theory of probabilities these circumstances are regarded as causes.

The *a posteriori* probability of an event or hypothesis is its probability deduced from events which have followed it, which events are regarded as its possible effects.

Theorem. — The *a priori* probability of an event is equal to the sum of the products obtained by multiplying the probability of each distinct cause by the probability that the event would occur on the hypothesis that the cause operates.

Let m be the whole number of separate cases. Let n of these cases be favorable to the first cause, n' to the second, &c. Their probabilities will then be $\frac{n}{m}, \frac{n'}{m}, \frac{n''}{m}$, &c. Also, let h of the n cases,

h' of the n' cases, &c., be favorable to the production of the event. Its probability, on the several hypotheses of the occurrence of each separate cause, will then be $\frac{h}{n}$, $\frac{h'}{n'}$, $\frac{h''}{n''}$, &c. Multiplying the corresponding fractions in the two series, and taking the sum of the products we obtain $\frac{h+h'+h''+\&c.}{m}$. But this is the probability of the event, because $h+h'+h''+$ &c. is the whole number of cases favorable to its production.

23. *Problem.* — Let E, E', E'', &c., represent a *portion* of the events which may occur on a certain trial, and let their respective probabilities be p, p', p'', &c. Suppose now that after I have estimated these probabilities, I am informed by a person who knows the result of the trial that one of the events E, E', E'', &c., has actually occurred. What ought to be their respective probabilities in my mind *after I have received this information?*

Let m be the whole number of possible cases; let n of these cases be favorable to E, n' to E', n'' to E'', &c. Then we have

$$p=\frac{n}{m}; \qquad p'=\frac{n'}{m'}, \text{ \&c.}$$

But when it is ascertained that one of the events E has actually occurred, the number of possible cases is *reduced* from m to $n+n'+n''+$ &c., those favorable to the production of some one of the events E. The probabilities of these events will therefore become

$$\frac{n}{n+n'+\&c.}, \qquad \frac{n'}{n+n'+\&c.};$$

that is, each of the former probabilities will be multiplied by the factor $\frac{m}{n+n'+n''+\&c.}$; or, *they will each be increased in the same ratio* to such a degree as to make their sum equal to unity.

Example. — On a table are fifty small boxes, of which three each contain a gold coin, seven a silver coin, thirteen a copper coin, and

twenty-seven nothing. The probability that one selected at random contains a gold coin is $\frac{3}{50}$, silver $\frac{7}{50}$, &c. But if by shaking the box it is found that it contains a coin, it will be known that one of the 23 boxes containing coins must have been selected. The probability that the box contains gold will then be increased to $\frac{3}{23}$, that it contains silver to $\frac{7}{23}$, that it contains copper to $\frac{13}{23}$.

24. *Problem.* — To find the probability of an event or hypothesis from both *a priori* and *a posteriori* circumstances.

Suppose that a possible event E, if it occur at all, must be preceded by *one* and *one only* of the causes or circumstances C, C', C'', &c. Let p, p', p'', &c., be the probabilities of C, C', C'', &c., before it is known whether the event E has occurred. Let q be the probability of E on the hypothesis that C occurs, q' its probability on the hypothesis that C' occurs, &c. The *a priori* probabilities of the compound events CE, $C'E'$, &c., will then be pq, $p'q'$, &c. If, now, it is ascertained that the event E has actually occurred, it becomes certain that *one* of these compound events has occurred; and, by § 23, their probabilities will be increased to

$$(1) \qquad \frac{pq}{pq + p'q' + \&c.}, \qquad \frac{p'q'}{pq + p'q' + \&c.}, \ \&c.$$

But, the event E being now certain, the probabilities of the causes C, C', &c., are the same as the probabilities of the compound events CE, $C'E'$, &c.; so that the above fractions represent the probabilities of the causes or hypotheses, deduced from all the circumstances in our knowledge, which bear on the question.

Example I. — A, handling a die, entertains a suspicion of $\frac{1}{3}$ that the die is so loaded that a six is as likely to be thrown as not. He throws it twice, and each time throws a six. What is the probability that his suspicion is correct? The solution is exhibited as follows: —

C	p	q	pq
die not loaded,	$\frac{2}{3}$	$\frac{1}{6^2} = \frac{1}{36}$	$\frac{1}{54}$,
die loaded,	$\frac{1}{3}$	$\frac{1}{2^2} = \frac{1}{4}$	$\frac{1}{12}$.

The probability that the die is loaded is therefore

$$\frac{\frac{1}{12}}{\frac{1}{54}+\frac{1}{12}} = \frac{9}{11}.$$

Had he thrown six a third time this probability would have increased to $\frac{27}{29}$.

Example II. — An urn contains 99 white balls, and 1 black one. An individual who falsifies once in ten times draws a ball at random, and asserts that it is black. What is the probability that he tells the truth?

The *a priori* probability that the black ball is drawn, and that he tells the truth respecting it, is $\frac{1}{100} \times \frac{9}{10}$. The *a priori* probability that a white ball is drawn, and that he falsifies, is $\frac{99}{100} \times \frac{1}{10}$. The probability that he tells the truth is therefore

$$\frac{\frac{9}{1000}}{\frac{9}{1000}+\frac{99}{1000}} = \frac{1}{12}.$$

NOTE.* — It is not necessary, as has been supposed by Mr. JOHN STUART MILL, that C and E should be connected by the *philosophical relation* of cause and effect. No such relationship has been supposed in our reasoning. The fraction pq does not, as Mr. MILL supposes, express the probability of E on the hypothesis that C is true, but the probability of the compound event CE. The groundlessness of the former opinion may be shown by solving the following problem. Find the *a priori* probability that it will rain to-morrow, considered as an effect of the planet Jupiter being inhabited or uninhabited.

Let p be the probability that Jupiter is inhabited, r the probability that it will rain to-morrow if Jupiter is inhabited, r also the probability of the same supposing Jupiter to be uninhabited.

By § 22 the probability required will be $pr + (1-p)r = r$.

This is the common-sense result, so that although we learn nothing new by bringing in irrelevant circumstances as causes, yet neither are we led into error by so doing.

Suppose now that to-morrow arrives, and it does rain; substituting in (1) we have for the probability that Jupiter is inhabited

$$\frac{pr}{pr+(1-p)r} = p.$$

If it does not rain, we have, by a corresponding supposition,

$$\frac{p(1-r)}{p(1-r)+(1-p)(1-r)} = p.$$

* Since this note was in type, we have found that MR. MILL has corrected this error.

THE METHOD OF INTEGRATION, &c.*

THE tables for integration at the end of the preceding example consist, besides the dates which limit the intervals of a column of arguments depending on the time from the beginning of the computation, or the number of intervals, a column containing the values of the differential coefficients for the several intervals, a column containing the sum of the series of values of the differential coefficients, and, in the case of the double integral, a column containing a second summed series, or a summing up of the numbers of the first summed series.

The following scheme represents the general form of the table for the double integral: —

Argument.	Principal Function. $f.$	I. Summed Series. $'f.$	II. Summed Series. $''f.$
$a-1\,\omega$	$f(a-1)$	$'f(a-\frac{1}{2})$	$''f(a-1)$
a	fa	$'f(a+\frac{1}{2})$	$''fa$
$a+1\,\omega$	$f(a+1)$	$'f(a+\frac{3}{2})$	$''f(a+1)$
$a+2\,\omega$	$f(a+2)$	$'f(a+\frac{5}{2})$	$''f(a+2)$
$a+3\,\omega$	$f(a+3)$	$'f(a+\frac{7}{2})$	$''f(a+3)$
$a+4\,\omega$	$f(a+4)$		$''f(a+4)$

ω being the interval, and a the time of beginning, or the date from which the intervals are counted forwards and backwards.

The terms of the two series are derived as follows: —

For the first series,

$$'f(a-\tfrac{1}{2})$$
$$'f(a+\tfrac{1}{2}) = fa + 'f(a-\tfrac{1}{2})$$
$$'f(a+\tfrac{3}{2}) = f(a+1) + 'f(a+\tfrac{1}{2})$$
$$'f(a+\tfrac{5}{2}) = f(a+2) + 'f(a+\tfrac{3}{2}).$$

* The reader of Professor ENCKE's treatise upon Mechanical Quadratures will perceive that the following explanation is not a connected translation of any particular part of that treatise, but a summary of the whole, generally made, for the sake of abbreviation, in my own language. — C. H. D.

For the second series,

$$\begin{aligned}
&''f(a-1)\\
&''fa \quad\quad = {}'f(a-\tfrac{1}{2}) + ''f(a-1)\\
&''f(a+1) = {}'f(a+\tfrac{1}{2}) + ''fa\\
&''f(a+2) = {}'f(a+\tfrac{3}{2}) + ''f(a+1)\\
&''f(a+3) = {}'f(a+\tfrac{5}{2}) + ''f(a+2)\\
&''f(a+4) = {}'f(a+\tfrac{7}{2}) + ''f(a+3), \text{ \&c.}
\end{aligned}$$

The comprehensive form of the single integral from the middle of the interval preceding the beginning $n = -\frac{1}{2}$ to the middle of any other interval is,

$$\begin{aligned}
\int_{-\frac{1}{2}}^{i+\frac{1}{2}} f(a+n\omega)\,dn = {}& {}'f\ (a+i+\tfrac{1}{2}) - {}'f\ (a-\tfrac{1}{2})\\
&+ \tfrac{1}{24}f'\ (a+i+\tfrac{1}{2}) - \tfrac{1}{24}f'\ (a-\tfrac{1}{2})\\
&- \tfrac{17}{5760}f'''(a+i+\tfrac{1}{2}) + \tfrac{17}{5760}f'''(a-\tfrac{1}{2})\\
&+ \tfrac{3867}{9676880}f^{v}\ (a+i+\tfrac{1}{2}) - \tfrac{3867}{9676880}f^{v}\ (a-\tfrac{1}{2})
\end{aligned}$$

It is evidently a matter of indifference what value is assigned to the term ${}'f(a-\frac{1}{2})$, in the series of the summed functions $'f$, since it is first added to every number in forming the series, and afterwards subtracted by the integration, as is seen in the above equation. It may either be taken $= 0$, or equal to the value of the integral up to the limit $n = -\frac{1}{2}$, if the new computation should join on to an earlier. If it be taken $= 0$, and the other terms depending on the limit ${}'f(a-\frac{1}{2})$ be taken together $= C_{-\frac{1}{2}}$, so that

$$C_{-\frac{1}{2}} = -\tfrac{1}{24}f'\ (a-\tfrac{1}{2}) + \tfrac{17}{5760}f'''(a-\tfrac{1}{2}) - \tfrac{3867}{9676880}f^{v}(a-\tfrac{1}{2})\ldots$$

then $C_{-\frac{1}{2}}$ becomes a constant for every later terminating limit. If $C_{-\frac{1}{2}}$ take the place of the term ${}'f(a-\frac{1}{2})$ in the series $'f$, then the first term of the series is

$${}'f(a+\tfrac{1}{2}) = C_{-\frac{1}{2}} + fa$$

instead of the above.

To join on the second series, any quantity whatever may be taken in the place of $''f(a-1)$, with which is derived $''fa = ''f(a-1)$

$+ C_{-\frac{1}{2}}$; $C_{-\frac{1}{2}}$ having taken the place of $'f(a-\frac{1}{2})$ in the series $'f$. In the first place, the value of the double integral up to $n = -\frac{1}{2}$, the beginning of the series, must be investigated. And for this purpose the quantities $''f, f, f''$, &c., must be interpolated for the argument $a - \frac{1}{2}\omega$, treating them as functions of $a - 1\omega$, and a. Let it be observed here, that, $''f$ being the second summed series, $'f$ are the first, and f the second differences of it; and that $f', f'', f''', f^{\text{IV}}$, &c., denote the first, second, third, fourth, &c., differences of the principal function f.

Professor ENCKE's form of interpolation gives, in the several series, the values,

$$\begin{aligned} ''f(a-\tfrac{1}{2}) - \tfrac{1}{8}f(a-\tfrac{1}{2}) + \tfrac{3}{128}f''(a-\tfrac{1}{2}) - \tfrac{5}{1024}f^{\text{IV}}(a-\tfrac{1}{2})\ldots. \\ f(a-\tfrac{1}{2}) - \tfrac{1}{8}f''(a-\tfrac{1}{2}) + \tfrac{3}{128}f^{\text{IV}}(a-\tfrac{1}{2}) \\ f''(a-\tfrac{1}{2}) - \tfrac{1}{8}f^{\text{IV}}(a-\tfrac{1}{2}) \\ f^{\text{IV}}(a-\tfrac{1}{2}) \end{aligned}$$

in which $''f(a-\frac{1}{2}) = \frac{1}{2}''f(a-1) + \frac{1}{2}''f a$ and $f(a-\frac{1}{2} = \frac{1}{2}f(a-1) + \frac{1}{2}f a$; in the same way all the other functions are arithmetical means. The numerical coefficients (as will be seen below) in the general expression of the double integral are for the first series 1, for the second series $+\frac{1}{12}$, for the third series $-\frac{1}{240}$, for the fourth series $+\frac{31}{60480}$; multiplying the respective series by these coefficients, and summing up the terms, gives for the value of the integral $n = \frac{1}{2}$,

$$''f(a-\tfrac{1}{2}) - \tfrac{1}{24}f(a-\tfrac{1}{2}) + \tfrac{17}{1920}f''(a-\tfrac{1}{2}) - \tfrac{367}{193536}f^{\text{IV}}(a-\tfrac{1}{2})\ldots$$

Since $''f(a-1)$ is wholly arbitrary, it can be put $= 0$; by this means

$$''f(a-\tfrac{1}{2}) = \tfrac{1}{2}''f(a-1) + \tfrac{1}{2}''f a,$$

or, from the above value of $''f a$,

$$= ''f(a-1) + \tfrac{1}{2}'f(a-\tfrac{1}{2}),$$

becomes $= \frac{1}{2}C_{-\frac{1}{2}}$, or according to the above value of $C_{-\frac{1}{2}}$,

$$''f(a-\tfrac{1}{2}) = -\tfrac{1}{48}f'(a-\tfrac{1}{2}) + \tfrac{17}{11520}f'''(a-\tfrac{1}{2}) - \tfrac{3867}{3870720}f^{v}(a-\tfrac{1}{2})\ldots$$

If this value is substituted in the integral, and it is reduced by means of the following simple equations, expressing the relations of interpolated differences,

$$\begin{aligned}
f\ (a-\tfrac{1}{2}) &= fa - \tfrac{1}{2}f'(a-\tfrac{1}{2}) \\
f''\ (a-\tfrac{1}{2}) &= \tfrac{1}{2}f''(a-1) + \tfrac{1}{2}f''a \\
f'''\ (a-\tfrac{1}{2}) &= f''a - f''(a-1) \\
f^{iv}\ (a-\tfrac{1}{2}) &= \tfrac{1}{2}f^{iv}(a-1) - \tfrac{1}{2}f^{iv}a \\
f^{v}\ (a-\tfrac{1}{2}) &= f^{iv}a - f^{iv}(a-1)
\end{aligned}$$

then the value of the integral for $n = -\frac{1}{2}$ may be written as follows:

$$-\tfrac{1}{24}fa + \tfrac{17}{5760}\{2f''a + f''(a-1)\} - \tfrac{3867}{967680}\{3f^{iv}a + 2f^{iv}(a-1)\}\ldots$$

This quantity must be subtracted throughout by reason of the first limit being taken $= -\frac{1}{2}$. If therefore we put in the second summed series, in the place of $''f(a-1)$, this quantity with opposite signs, or

$$C'_{-\frac{1}{2}} = +\tfrac{1}{24}fa - \tfrac{17}{5760}\{2f''a + f''(a-1)\} + \tfrac{3867}{967680}\{3f^{iv}a + 2f^{iv}(a-1)\}\ldots.$$

this subtraction will be made once for all.

With these substitutions the complete scheme for $n = -\frac{1}{2}$ taken as the first limit is,

Argument.	f Principal Function.	$'f$ I. Summed Series.	$''f$ II. Summed Series.
$a-1\,\omega$	$f(a-1)$	$C_{-\frac{1}{2}}$	$C'_{-\frac{1}{2}}$
a	fa	$'f(a+\frac{1}{2})$	$''fa$
$a+1\,\omega$	$f(a+1)$	$'f(a+\frac{3}{2})$	$''f(a+1)$
$a+2\,\omega$	$f(a+2)$	$'f(a+\frac{5}{2})$	$''f(a+2)$
$a+3\,\omega$	$f(a+3)$	&c.	$''f(a+3)$
	&c.		&c.

The second term of the second series being now

$$''f\,a = C'_{-\frac{1}{2}} + C_{-\frac{1}{2}}.$$

For every subsequent limit, $a + (i + n'')\,\omega$, the expression is

$$\iint f(a + n\,\omega)\,dn^2 = ''f(a + i + n'') + \tfrac{1}{12} f(a + i + n'') - \tfrac{1}{240} f''(a + i + n'') + \tfrac{31}{60480} f^{\mathrm{IV}}(a + i + n'') \ldots\ldots$$

So that for every n'' the several functions must be interpolated as if the values actually occurring in the summed series $''f$ were functions of $a + i\,\omega$.

In the special case of $n'' = \frac{1}{2}$ the interpolation takes place into the middle of the interval between the terms, as shown in the above example of $n = -\frac{1}{2}$.

The integral for this value of n'' is,

$$\iint_{-\frac{1}{2}}^{i+\frac{1}{2}} f(a + n\,\omega)\,dn^2 = ''f(a + i + \tfrac{1}{2}) - \tfrac{1}{24} f(a + i + \tfrac{1}{2}) + \tfrac{17}{1920} f''(a + i + \tfrac{1}{2}) - \tfrac{367}{193536} f^{\mathrm{IV}}(a + i + \tfrac{1}{2}) \ldots$$

in which by $''f(a + i + \frac{1}{2})$ is understood the arithmetical mean between $''f(a + i)$ and $''f(a + i + 1)$, and so with all the other functions f, f'', f^{IV}.

It need be mentioned merely, that, in both cases, the generally indispensable factor ω^2 for the double integral is considered as comprised in the functions f, so that, to derive the first integral from the same table of functions, it is necessary, on this account, to divide by ω.

In order to illustrate by an example the formation of the series at the beginning, I have introduced, from Professor ENCKE's treatise, the following table, of which the table at the end of the previous computation is the continuation, the intervening part of the series being omitted.

0h P. M. T.	Argument	$42^2 \frac{d\mu}{dt}$	^{I}f	^{II}f
		"	"	"
1809, Sept. 17	$a - 3\omega$	+6.57219		
Oct. 29	$a - 2\omega$	+5.82592	. . . .	. . . . .
Dec. 10	$a - \omega$	+5.15193	. . . .	. . . . .
1810, Jan. 21	a	+4.55359	+ 0.02489	+ 0.18914
March 4	$a + \omega$	+4.01802	+ 4.57848	+ 0.21403
April 15	$a + 2\omega$	+3.53914	+ 8.59650	+ 4.79251
May 27	$a + 3\omega$	+3.10646	+12.13564	+ 13.38901
July 8	$a + 4\omega$	+2.71163	+15.24210	+ 25.52465
Aug. 19	$a + 5\omega$	+2.34609	+17.95373	+ 40.76675
Sept. 30	$a + 6\omega$	+2.00418	+20.29982	+ 58.72048
Nov. 11	$a + 7\omega$	+1.68040	+22.30400	+ 79.02030
Dec. 23	$a + 8\omega$	+1.36911	+23.98440	+ 101.32430
1811, Feb. 3	$a + 9\omega$	+1.06719	+25.35351	+ 125.30870
March 17	$a + 10\omega$	+0.77184	+26.42070	+ 150.66221
April 28	$a + 11\omega$	+0.48183	+27.19254	+ 177.08291
				+ 204.27545
. . . . .	. . . . .	. . .	. . . .	. . . . .
1836, May 22	$a + 229\omega$	—0.7115		—4802.3339
July 3	$a + 230\omega$	—1.3811	—19.5017	—4821.8356
Aug. 14	$a + 231\omega$	—1.8398	—20.8828	—4842.7184
Sept. 25	$a + 232\omega$	—2.0960	—22.7226	—4865.4410
Nov. 6	$a + 233\omega$	—2.1546	—24.8186	—4890.2596
Dec. 18	$a + 234\omega$	—2.0412	—26.9732	—4917.2328

The integral $$\Delta M = \iint . \frac{d\mu}{dt} . dt^2$$

will serve for an example of the double integration, in which μ is the mean daily sidereal motion of Vesta, and $\frac{d\mu}{dt}$ is the differential coefficient, relative to the time, of the perturbation of the μ of Vesta caused by the direct attraction of Jupiter.

By means of the first integral $\int \frac{d\mu}{dt} dt$, is obtained the true μ affected by the perturbations. The second gives the mean anomaly resulting from the perturbations of μ. The unit in $\frac{d\mu}{dt}$ is the mean day, and the interval of time has been taken equal to 42 days. The beginning of both integrals falls upon January 0, 1810. In order to obtain ΔM immediately

$$\omega^2 . \frac{d\mu}{dt} = 1764 \frac{d\mu}{dt}$$

has been put as the function f, and the values of $f(a + n\omega)$ have been computed for

$a - 3\omega$, $a - 2\omega$, a, $a + \omega$, $a + 2\omega$, &c..... $a + 233\omega$, $a + 234\omega$,

in which $a =$ January 0, 1810, $+ \frac{1}{2}\omega =$ January 21, 1810.

It will be observed that the constants $C_{-\frac{1}{2}}$, $C'_{-\frac{1}{2}}$, have taken the place in the table of $'f(a - \frac{1}{2})$ and $''f(a - 1)$; in order to obtain these constants we have

$$f'\ (a - \tfrac{1}{2}) = -\ 0.59834$$
$$f''\ (a - 1) = +\ 0.07565$$
$$f''\ a = +\ 0.06277$$
$$f'''\ (a - \tfrac{1}{2}) = -\ 0.01288$$

It is not necessary to go farther back than f''', because the higher differences are insensible, and the terms are wanting in the beginning of the series to obtain them, except when two computations made from different elements meet in this place. By means of the above values,

$$C_{-\frac{1}{2}} = +\ 0.02493 - 0.00004 = +\ 0.02489$$
$$C'_{-\frac{1}{2}} = +\ 0.18973 - 0.00059 = +\ 0.18914.$$

The argument for January 13, 1811, $= a + (8 + \frac{1}{2}\omega)$, whence

$$''f\ (a + \tfrac{17}{2}) = +\ 137.98545\ .\ 5$$
$$-\ \tfrac{1}{24}\ f\ (a + \tfrac{17}{2}) = -\ \ 0.05075\ .\ 6$$
$$+\ \tfrac{17}{1920} f''\ (a + \tfrac{17}{2}) = +\ \ 0.00007\ .\ 1$$

and $\Delta M = +\ 137.93477$ is the amount of ΔM from January 0, 1810, to January 13, 1811.

Again, the argument for July 3, 1836, is $(a + 230\omega)$, and

$$''f\ (a + 230) = -\ 4821.8356$$
$$+\ \tfrac{1}{12}\ f\ (a + 230) = -\ \ 0.1150\ .\ 9$$
$$-\ \tfrac{1}{240} f''\ (a + 230) = +\ \ 0.0008\ .\ 8$$

whence $\Delta M = -\ 4821.9498$ is the double integral of ΔM from January 0, 1810, to July 3, 1836.

We may employ the same functions to furnish an example of the single integral, remembering, as before mentioned, that when a single integral is derived from a table of functions comprising the factor ω^2, it must be divided by ω.

For the first argument we have

$$
\begin{aligned}
{}'f\ (a + \tfrac{17}{2}) &= + 25.35351 \\
+ \tfrac{1}{24}\ f'\ (a + \tfrac{17}{2}) &= - \ 0.01258 \\
- \tfrac{17}{5760} f''' (a + \tfrac{17}{2}) &= + \ 0.00001 \\
42\, \Delta \mu &= + 25.34094 \\
\Delta \mu &= + \ 0.603356
\end{aligned}
$$

from January 0, 1810, to January 13, 1811.

And for the argument $(a + 232\,\omega)$ we have

$$
\begin{aligned}
{}'f\ (a + 232\,\omega) &= - 23.7706 \\
- \tfrac{1}{12}\ f'\ (a + 232\,\omega) &= - \ 0.0131\,.\,2 \\
+ \tfrac{11}{7200} f''' (a + 232\,\omega) &= - \ 0.0001 \\
42\, \Delta \mu &= - 23.7838\,.\,2 \\
\Delta \mu &= \ \ 0.5662\,.\,8
\end{aligned}
$$

from January 0, 1810, to September 25, 1836.

This being the first time that the single integral

$$\int_{-\frac{1}{2}}^{i} f(a + n\,\omega)\, dn,$$

in which n is a whole number, has been introduced, it may be well to notice that the numerical coefficients of the different series are changed from $+ 1, + \frac{1}{24}, - \frac{17}{5760}$, to $+ 1, - \frac{1}{12}, + \frac{11}{7200}$.

Should we desire to take immediately from the table the first and second integrals for any other time, as, for example, for February 10, 1811, or for the argument $a + (9 + \frac{1}{6})\,\omega$, we must interpolate for $42\, \Delta \mu$ as follows:—

$$'f\ (a+9+\tfrac{1}{6}) = {}'f\ (a+9+\tfrac{1}{2}-\tfrac{1}{3}) = +\ 26''.09809$$
$$f'\ (a+9+\tfrac{1}{6}) = f'\ (a+9+\tfrac{1}{2}-\tfrac{1}{3}) = -\ 0.29740$$
$$f'''(a+9+\tfrac{1}{6}) = f'''(a+9+\tfrac{1}{2}-\tfrac{1}{3}) = -\ 0.00175;$$

and for ΔM,

$$''f\ (a+9+\tfrac{1}{6}) = +\ 154.99961$$
$$f\ (a+9+\tfrac{1}{6}) = \quad 1.01754$$
$$f''(a+9+\tfrac{1}{6}) = \quad 0.00636,$$

from which we should obtain

$$42\,\Delta\mu = +\ 26.09809 - 0.01239\,.\,2 + 0.00000\,.\,5$$
$$= +\ 26.08570$$
$$\Delta\mu = +\ 0.621088 \text{ from Jan. 0, 1810, to Feb. 10, 1811;}$$
and $$\Delta M = +\ 154.99961 + 0.08479\,.\,5 - 0.00002\,.\,6$$
$$= +\ 155.08438 \text{ from Jan. 0, 1810, to Feb. 10, 1811,}$$

agreeing with the interpolation from the previous values.

When the amount of the remaining perturbations is computed, the actual value of μ for January 0, 1810, is lastly to be added to $\Delta\mu$, and so also the value which M would have without the perturbations at each time is to be added to ΔM. The first sum may be denoted by μ_0, and the form of the last will be $M_0 + (n+\frac{1}{2})\,\omega\,\mu_0$. It would only be an unnecessary addition to the calculation to put in the place of the constant $C_{-\frac{1}{2}}$ the value $C_{-\frac{1}{2}} + 42\,\mu_0$, and in the place of $C'_{-\frac{1}{2}}$ the value $C'_{-\frac{1}{2}} + M_0 - \frac{1}{2}\,\omega\,\mu_0$, for the purpose of obtaining the whole result immediately from the table.

Finally, the connection of the two parts of the above table may be shown by computing the value of the formula

$$M = L - \pi + \mu\,(t - T)$$

according to the precept.

$$L_0 = 105^\circ\ 53'\ 15''.63$$

$$\iint_{t'=\text{Jan. 0, 1810.}}^{t''=\text{June 12, 1836.}} \frac{d\mu}{dt} \cdot dt^2 = -1\ 20\ 12.03$$

$$\int_{t'=\text{Jan. 0, 1810.}}^{t''=\text{June 12, 1836.}} \frac{dL}{dt} \cdot dt = -0\ 17\ 51.62$$

$$\mu(t-T) = 105\ \ 5\ 46.21$$

$$L = 209\ 20\ 58.2 + 360^\circ = 569^\circ\ 20'\ 58''.2$$

$$\pi_0 = 249\ 48\ 26.9$$

$$\int_{t'=\text{Jan. 0, 1810.}}^{t''=\text{June 12, 1836.}} \frac{d\pi}{dt} \cdot dt = 0\ 19\ 29.45$$

$$\pi = 250\ \ 7\ 56.4 \qquad 250^\circ\ 7'\ 56''.4$$

$$M = L - \pi = 319\ 13\ 1.8$$

And by the elements answering to the date June 12, 1836.

$$M = 319\ 13\ 1.8$$

$$\pi = 250\ \ 7\ 56.4$$

$$L = M + \pi = 209\ 20\ 58.2$$

ON THE DEPENDENCE OF NAPIER'S RULES.

By Rev. Anthony Vallas, Phil. Dr., New Orleans, La.

The five parts of a right-angled spherical triangle being placed on the circumference of a circle, Napier's Rules are as follows: —

Rule I. *The sine of the middle part equals the product of the cosines of the opposite parts.*

Rule II. *The sine of the middle part is equal to the product of the tangents of the adjacent parts.*

It must be remembered that, instead of the hypothenuse and the two acute angles, their complements are used, the right angle not being counted among the parts.

That the second of these rules may be deduced from the first has been shown by Mr. SAFFORD, in No. I. Vol. I. We are going to show the same in the following way. Apply the first rule to the three parts α, α', ω', and then to α, α', ω. Eliminate one of the common parts, for instance α, and the result, properly transformed, will give the second rule. From

$$\sin\alpha = \cos\alpha' \cos\omega', \qquad \sin\alpha' = \cos\alpha \cos\omega,$$

we get $$\sin^2\alpha' = (1 - \cos^2\alpha' \cos^2\omega') \cos^2\omega;$$

hence $$\frac{1}{\cos^2\omega} = \frac{1}{\sin^2\omega} - \cot^2\alpha' \cos^2\omega',$$

$$1 + \tan^2\omega = 1 + \cot^2\alpha' - \cot^2\alpha' \cos^2\omega',$$

from which follows

$$\sin^2\omega' = \tan^2\alpha' \tan^2\omega \qquad \therefore \sin\omega' = \tan\alpha' \tan\omega;$$

or NAPIER'S Rule II.

In order to find the second rule from the first, take the three adjacent parts α, μ, α', and μ, α', ω', and eliminate μ from the two equations

$$\sin\mu = \tan\alpha \tan\alpha', \qquad \sin\alpha' = \tan\mu \tan\omega'.$$

Since

$$\tan\mu = \frac{\sin\mu}{\sqrt{1 - \sin^2\mu}},$$

we get

$$\sin\alpha' = \frac{\tan\alpha \tan\alpha' \tan\omega'}{\sqrt{1 - \tan^2\alpha \tan^2\alpha'}}; \text{ or}$$

$$1 - \tan^2\alpha \tan^2\alpha' = \frac{\tan^2\alpha \tan^2\omega'}{\cos^2\alpha'}.$$

Replacing the tangents by their equivalents in cosines, we have

$$\frac{1}{\cos^2\alpha \cos^2\alpha' \cos^2\omega'} - \frac{1}{\cos^2\alpha' \cos^2\omega'} - \frac{1}{\cos^2\alpha} = 0; \text{ or}$$

$$1 - \cos^2\alpha - \cos^2\alpha' \cos^2\omega' = 0; \text{ or}$$

$$\sin^2\alpha = \cos^2\alpha' \cos^2\omega'; \qquad \therefore \sin\alpha = \cos\alpha' \cos\omega',$$

which is NAPIER'S Rule I.

ON A TRANSFORMATION OF THE DERIVATIVE OF ANY POWER OF A VARIABLE.

By Rev. George Clinton Whitlock, LL.D.,
Professor of Natural Sciences in the University of Victoria College, Coburg, Canada West.

THEOREM. — *Any power of a variable, regarded as a factor of any power of the same, may be carried from under the sign of derivation, by adding algebraically the product formed on multiplying its exponent into the variable having an exponent less by unity than that of the undecomposed power;*

or $$D(x^n) = x^r\, D(x^{n-r}) + r\, x^{n-1},$$

whatever n *and* r *may be, plus or minus, whole or fractional, or imaginary.*

DEMONSTRATION. — From the laws of exponents in general, whatever n may be, plus or minus, whole or fractional, or imaginary, we have

$$x^n = x\, x^{n-1};$$

$$\therefore\ (x+h)^n = (x+h)\,(x+h)^{n-1},$$

by changing x into $(x+h)$; consequently, subtracting member by member and separating the factors x, h,

$$(x+h)^n - x^n = x\,[(x+h)^{n-1} - x^{n-1}] + h\,(x+h)^{n-1},$$

and, dividing by h,

$$\frac{(x+h)^n - x^n}{h} = x\,\frac{(x+h^{n-1} - x^{n-1}}{h} + (x+h)^{n-1};$$

therefore, passing to the *limit*, that is, taking the values of the terms when h reduces to zero,

$$\lim \frac{(x+h)^n - x^n}{h} = x \lim \frac{(x+h)^{n-1} - x^{n-1}}{h} + x^{n-1}, \text{ or}$$

(1) $$D(x^n) = x\, D(x^{n-1}) + x^{n-1};$$ i. e.

the derivative of x^n is equal to x times the derivative of x^{n-1} plus

x^{n-1}; or a factor x of any power x^n may be carried from under the sign D, of derivation, by adding x^{n-1}.

Therefore by (1)

$$D(x^{n-1}) = x\,D(x^{n-2}) + x^{n-2},$$

and by replacing $D(x^{n-1})$ by this value, (1) becomes

$$D(x^n) = x^2\,D(x^{n-2}) + 2\,x^{n-1};$$

and in like manner this form may be changed into

$$D(x^n) = x^3\,D(x^{n-3}) + 3\,x^{n-1};$$

and in general, the same operation being repeated r times, we have

(2) $$D(x^n) = x^r\,D(x^{n-r}) + r\,x^{n-1};$$ i. e.

any number, r, of factors, x, may be carried from under the sign D by adding $r\,x^{n-1}$.

From (2), dividing by x^r, we have

$$x^{-r}\,D(x^n) = D(x^{n-r}) + r\,x^{n-r-1}, \text{ or}$$

$$D(x^{n-r}) = x^{-r}\,D(x^n) - r\,x^{n-r-1}, \text{ or}$$

(3) $$D(x^m) = x^{-r}\,D(x^{m+r}) - r\,x^{m-1},$$

putting $n - r = m$, which may be any number whatever, since n is any number; hence (3), with proper observance of sign, for the separation of reciprocal factors, $x^{-r} = \frac{1}{x^r}$, we have the same rule as above, (1).

Again (2, 3), ϱ and r being any whole numbers, plus or minus,

(4) $$D(x^n) = x^{\varrho r}\,D(x^{n-\varrho r}) + \varrho\,r\,x^{n-1};$$ i. e.

a factor x^r being separated, a power of this factor, $(x^r)^\varrho = x^{\varrho r}$, will be separated by multiplying r by ϱ.

It follows that

$$(5) \qquad D(x^n) = x^{\frac{r}{\nu}} D\left(x^{n-\frac{r}{\nu}}\right) + \frac{r}{\nu} x^{n-1},$$

since by the last, multiplying $\frac{r}{\nu}$ by ν, we have

$$D(x^n) = x^r D(x^{n-r}) + r x^{n-1},$$

which, as necessary and sufficient, accords with (2). Therefore (2), (3), (5), the theorem is proved, since whatever is found for real exponents must be extended to those that are imaginary.

Scholium. — The deduction of (3) and (5) from (2) is in accordance with the general theory of exponents, as indicating, whether plus or minus, whole or fractional, or imaginary, the number of times the quantity to which they are affixed, must be taken as a factor, or so regarded, in executing any operation. Thus x^r is x taken r times as a factor, and theory requires that x^{-r} be regarded as x taken $-r$ times as factor, and so $x^{\frac{1}{\nu}}$ is x taken $\frac{1}{\nu}$ times as factor, and $x^{\frac{r}{\nu}} = \left(x^{\frac{1}{\nu}}\right)^r$ is $x^{\frac{1}{\nu}}$ taken r times as factor, or x taken $r \cdot \frac{1}{\nu}$ or $\frac{r}{\nu}$ times as factor. It is true that here, as elsewhere, we have to extend the signification of the phrase, "*number of times*," that is, "*number*" must be regarded, not only as *continuous* (of any magnitude between integers), but as plus, or minus, or imaginary. And it is by this extension, *precisely*, that Analysis has its *power*, or better, an *existence*.

Corollary. — Whatever n may be, from the theorem we have

$$D(x^n) = x^n D(x^0) + n x^{n-1};$$

but $$D(x^0) = \lim \frac{(x+h)^0 - x^0}{h} = \lim \frac{1-1}{h} = \lim \frac{0}{h} = \lim 0 = 0;$$

$$\therefore D(x^n) = n x^{n-1}; \text{ i. e.}$$

the derivative of any power of a variable is found by diminishing the exponent by unity and multiplying by the original exponent.

EXPOSITION OF THE PROCESS OF MATHEMATICAL DEVELOPMENT.

By JOHN PATERSON, A. M., Albany, N. Y.

1. THE measure of an effect is adopted as the measure of its cause; and in mathematical investigations, the effect is the measure of the force that produces it. A constant force produces equal effects in equal times; but when the force is variable, the results in equal successive times are unequal. Beginning with zero, if a force increase by a regular law during a given time, and then remain constant with its ultimate value during an equal time, the effect in the latter interval of time will be some determinate number m of times its effect in the former interval.

The immediate product of a force may be itself a force, and in its turn produce a third force, and so on for several descending ranks or orders of generated and generating forces, decreasing in intension but increasing in extension, until finally arriving at the order zero, which will be the phenomenal effect, the measure of which in space and time is the *base line* of reference for the determination of the genetical operations of the subordinated system of forces under consideration. For instance, the immediate effect of the force of gravitation in the extent of space for which it may be regarded as a constant or uniform activity, is the product or genesis of a uniform velocity; and this (unresisted) velocity causes the body in which it is developed to describe a distance which increases as the square of the time, which distance is the phenomenal effect, and its measure for the first unit of time occupied in the genetical process is the unit measure of reference.

2. Let φ^n be a constant force of the nth order; it generates force φ^{n-1} of the $(n-1)$th order uniformly in time. As φ^{n-1} increases

uniformly, as in the simple ratio of the time, it generates force φ^{n-2} of the $(n-2)$th order with a uniformly accelerated energy, or in duplicate ratio of the time; so that the increase of the latter force increases uniformly, or the force itself is duplicately accelerated. In its turn φ^{n-2} generates force φ^{n-3} of the $(n-3)$th order with a duplicately accelerated energy, so that the increase of the increase of the latter force increases uniformly, or the force itself is triplicately accelerated. This process of augmented acceleration continues through successive descending orders, until arriving at the force $\varphi^{n-n} = \varphi^0$ of the order zero, the phenomenal effect, which is measurable in space and time, and serves to determine the measures of the several generating forces.

For greater simplicity, the phenomenal effect produced or generated in a unit of time 1_t may be adopted as the unit of effect in the system of operations. Beginning at zero with the genetical operations, the measure of the phenomenal effect in the first unit $1'_t$ of time is denoted by 1_l the linear unit, this being the distance that the unit of mass 1_μ is assumed to be carried in that time by the force $\varphi^{n-n+1} = \varphi'$ of the first order, as a general and simple representative of the unit measure of an operation. This definition requires that unity, expressed in the coefficient 1, shall be the measure of the force φ', and indeed also of each member of the descending series φ^n, φ^{n-1}, φ^{n-2}, ... for the first unit $1'_t$ of time occupied in the genetical process; a requirement that is obviously legitimately based upon the logical admeasurement of causes by their effects, and is indispensable for the determination of the effects that will arise in the succeeding units of time. All the forces from φ^{n-1} to φ' inclusive are variable; each increases from 0 to some determinate magnitude, $m\varphi^{n'}$ in the first unit of time $1'_t$, and then remains a constant or uniform generator during the second unit of time $1''_t$ with the ultimate value it has acquired at the termination

of the first unit of time. The ratio of the effect of each force thus conditioned, during the second interval $1_t''$ of time, to its effect during the equal interval $1_t'$ of time occupied in its own genesis or growth, is the very relation to be established.

3. The constant force φ^n generates equal increments of force φ^{n-1} in successive equal intervals of time $1_t'$, $1_t''$, $1_t'''$, etc. The force φ^{n-1} therefore increases uniformly from 0 during the first unit of time $1_t'$; and if then remaining constant with its acquired value during the second unit of time $1_t''$, it will generate some definite number m of times the amount of force φ^{n-2} that was generated by it during the equal time $1_t'$ of its own increase or growth; that is, if φ^{n-1}, with a uniformly increasing energy, has generated $1\,\varphi^{n-2}$ in $1_t'$, then with its ultimate energy remaining constant, it will generate $m\,\varphi^{n-2}$ during $1_t''$. Then φ^{n-1} has increased from 0 to the value $m\,\varphi^{n-1}$ during the time $1_t'$; and during this time $1_t'$ of its increase, it has generated the mth part as much as will be its effect as a constant generator during the equal time $1_t''$.

4. As φ^{n-1} increases uniformly, its value at the expiration of the first semi-unit $\frac{1}{2}'.1_t'$ of time will be $\frac{m}{2}\varphi^{n-1}$; with this value remaining constant, it would generate $\frac{m}{2}\varphi^{n-2}$ in a unit of time, and consequently it does generate $\frac{m}{4}\varphi^{n-2}$ in the second semi-unit $\frac{1}{2}''.1_t'$ of time, the mth part of which amount is $\frac{1}{4}\varphi^{n-2}$ the effect of $\frac{m}{2}\varphi^{n-1}$ during the time $\frac{1}{2}'.1_t'$ of its own genesis. During the second semi-unit $\frac{1}{2}''.1_t'$ of the first unit $1_t'$ of time, a second equal increment $\frac{m}{2}\varphi^{n-1}$ is generated by the primitive generator $1\,\varphi^n$, and this increment likewise generates $\frac{1}{4}\varphi^{n-2}$ during the time of its own genesis. The sum of these effects, estimated in and for the first unit of time, is conditioned to be unity; hence the equation $\left(\frac{1}{4}+\frac{m}{4}+\frac{1}{4}\right)\varphi^{n-2}=1\,\varphi^{n-2}$.

In the equations

$$\begin{aligned}
&\tfrac{1}{2}'.1_t' \;+\tfrac{1}{2}''.1_t' \;= 1_t';\\
&\tfrac{1}{2}\varphi_1^{n-1} + \tfrac{1}{2}\varphi_2^{n-1} = 1\,\varphi^{n-1},\\
&\tfrac{m}{2}\varphi_1^{n-1} + \tfrac{m}{2}\varphi_2^{n-1} = m\,\varphi^{n-1},\\
&\left.\begin{aligned}\tfrac{1}{4}\varphi_1^{n-2} + \tfrac{m}{4}\varphi^{n-2}\\ + \tfrac{1}{4}\varphi_2^{n-2}\end{aligned}\right\} = 1\,\varphi^{n-2},
\end{aligned}$$

the terms of the left-hand member of the first are the constant equivalents of the corresponding varying terms of the second, and those of the third exhibit the effects generated by their correspondents of the second, the measures of these effects being confined to what shall abide throughout in the first unit of time. The third equation gives $m = 2$, which value converts the immediately preceding one into $\frac{2}{2}\varphi^{n-1} + \frac{2}{2}\varphi^{n-1} = 2\,\varphi^{n-1}$, and proves that when a force increases uniformly from 0 during a given time, and then remains constant during an equal time, its effect during the second interval will be twice that during the first interval. Thus the *successive* and *synchronous* unit measures of a force of the second rank (from its constant generator) are mutually convertible by the factor 2.

5. During the time $1_t'$, the entire force φ^{n-2} increases from 0 in duplicate ratio; and consequently its effect in the time $1_t''$, with its acquired or ultimate value in $1_t'$ remaining constant, will be some number m' of times its effect in the time $1_t'$; this ultimate value of φ^{n-2} is made up of the sum of the ultimate values of the partial forces of which $m'\,\varphi^{n-2}$ is constituted, and therefore the transition from the first to the second of the following equations must be shown.

$$\begin{aligned}
&\tfrac{1}{2}'.1_t' \;+\tfrac{1}{2}''.1_t' = 1_t';\\
&\left.\begin{aligned}\tfrac{1}{4}\varphi_1^{n-2} + \tfrac{2}{4}\varphi^{n-2}\\ + \tfrac{1}{4}\varphi_2^{n-2}\end{aligned}\right\} = 1\,\varphi^{n-2},\\
&\left.\begin{aligned}\tfrac{m'}{4}\varphi_1^{n-2} + 2\tfrac{m'}{4}\varphi^{n-2}\\ + \tfrac{m'}{4}\varphi_2^{n-2}\end{aligned}\right\} = m'\,\varphi^{n-2}.
\end{aligned}$$

The first and third terms of the left-hand members correspond as constant to variable as in the preceding case, but this remark does not directly apply to the second terms.

The force $\frac{m'}{4}\varphi_1^{n-2}$ would generate $\frac{m'}{4}\varphi^{n-3}$ in a unit of time, and therefore does generate $\frac{m'}{8}\varphi^{n-3}$ in the time $\frac{1}{2}''.1_t'$, the m'th part of which is $\frac{1}{8}\varphi^{n-3}$ the amount it has generated during the time $\frac{1}{2}'.1_t'$ of its own genesis, and is likewise the amount generated by $\frac{m'}{4}\varphi_2^{n-2}$ in the time $\frac{1}{2}''.1_t'$ of its own genesis.

The force $2\frac{m'}{4}\varphi^{n-2}$ would generate $2\frac{m'}{4}\varphi^{n-3}$ in a unit of time, or $2\frac{m'}{8}\varphi^{n-3}$ in a semi-unit of time; but whereas the forces $\frac{m'}{4}\varphi_1^{n-2}$ and $\frac{m'}{4}\varphi_{\frac{1}{2}}^{n-2}$ each increase in m'plicate ratio during their genesis, and therefore have the value of their successive effect divided by m' to get that of their synchronous effect, the force $2\frac{m'}{4}\varphi^{n-2}$ increases in duplicate ratio from 0 to this, its ultimate value, during the time $\frac{1}{2}''.1_t'$, being itself generated by the constant $\frac{2}{2}\varphi_1^{n-1}$ in this time, so that the preceding result must be divided by 2 to give $\frac{2}{2}\frac{m'}{8}\varphi^{n-3}$ as the amount generated by $2\frac{m'}{4}\varphi^{n-2}$ during its own genesis in the time $\frac{1}{2}''.1_t'$.

As the result in and for the time $1_t'$ must be unity, the collected effects constitute the equation $\left(\frac{1}{8}+\frac{2}{2}\frac{m'}{8}+\frac{m'}{8}+\frac{1}{8}\right)\varphi^{n-3}=1\,\varphi^{n-3}$, which gives $m'=3$, and converts the last preceding equation into

$$\left.\begin{array}{l} \frac{1}{2}'.1_t' \quad + \frac{1}{2}''.1_t' = 1_t'\,; \\ \frac{3}{4}\varphi_1^{n-2} + 2\,.\,\frac{3}{4}\varphi^{n-2} \\ \qquad\quad + \frac{3}{4}\varphi_2^{n-2} \end{array}\right\} = 3\,\varphi^{n-2}.$$

Thus the successive and synchronous measures of a force of the third rank are mutually convertible by the factor 3.

[To be Continued.]

Mathematical Monthly Notices.

Résumé de Leçons de Géométrie Analytique et de Calcul Infinitésimal, Comprenant sur la Trigonométrie, sur l'Expression des Lieux Géométriques par leurs Équations, sur le Calcul différential et sur les Calcul intégral, l'Exposition des Connaissances nécessaires aux Ingénieurs pour l'Intelligence de la Mécanique rationnelle, de l'Hydraulique et de la Théorie dynamique des Machines. Par J. B. BELANGER, Ingénieur en Chef des Ponts et Chaussées, Professeur de Mécanique à l'Ecole impériale Polytechnique et à l'Ecole Centrale des Arts et Manufactures. Seconde Édition. Paris: Mallet-Bachelier. 1859. pp. 295. Tigr. 104.

To this full title-page but little need be added to give a good idea of the work before us. It is simply intended to supply all the elementary knowledge in Trigonometry, Analytic Geometry, and the Calculus, which the student requires to read the course of rational mechanics as taught in the French schools of engineering. The subjects are clearly treated, and the only peculiarity we notice is, that the subject of Trigonometry is made to depend upon the theory of projections.

Théorie des Fonctions doublement Périodiques et, en particulier des Fonctions Elliptiques. Par M. BRIOT et M. BOGUET. Paris: Mallet-Bachelier. 1859. pp. 342.

The subjects discussed in this book are sufficiently indicated by the title. The method of discussion, which, in fact, involves a completely new "theory of functions," is one, probably, almost *wholly unknown* to American students. The authors have developed and extended the theory of imaginaries named by the illustrious CAUCHY "*les quantités géométriques;*" of which a full discussion may be found in his "*Exercices d'Analyse et de Physique Mathématique*," Tome IV.; and of which some application is made by M. V. PUISEUX in his "*Recherches sur les Fonctions Algébriques*," in LIOUVILLE'S "*Journal de Mathématiques*," Tome XV., p. 365. This view of $\sqrt{-1}$ is founded on the theories of ARGAND, MOUREY, etc. CAUCHY'S "*geometric quantity*," which he denotes by r_θ is algebraically expressed by $x + y\sqrt{-1} = r(\cos\theta + \sin\theta \cdot \sqrt{-1}) = r\,e^{\theta\sqrt{-1}}$, and these quantities may be added, multiplied, etc. just as *coplanar quaternions* would be. The *modulus* r corresponds to the *tensor*, and the *argument* θ to the *angle* of the quaternion. (See p. 133 of the present volume of the Monthly, equation (34).) We have no doubt that this book, especially if read with the two articles we have cited, presents an excellent opportunity to form an acquaintance with the recent progress of the higher branches of analysis among French mathematicians, just as SALMON'S books and BOOLE'S "*Differential Equations*" (heretofore noticed in the Monthly) do for the widely different paths of discovery pursued of late, more especially by British investigators. We scarcely need add, that the *Elliptic Transcendants* appear here in a totally different dress from that given them by their first great expounder, LEGENDRE; or even in the refined analysis of ABEL, although, as with the latter, the inverse function forms the base of the investigations. A second volume is promised, to contain the requisite tables, with practical applications, etc. — B.

Leçons sur les Coördonnées Curvilignes et leurs diverses Applications. Par G. LAMÉ. Paris: Mallet-Bachelier. 1859. pp. 368.

In some of his other books, the author has introduced, to a greater or less extent, the use of these peculiar coördinates invented by himself. If we conceive a family of similar ellipsoids depending on the variable parameter α, such that when α varies we pass from one ellipsoid to the next, etc.; and in the same way a family of one-sheeted hyperboloids depending on a parameter β, and one of two-sheeted hyperboloids depending on γ; then α, β, and γ will be *curvilinear coördinates* of the common intersection-points of these three surfaces. This and similar

systems have been found convenient, if not necessary, in many physical investigations, especially those involving the consideration of *level surfaces*, *isothermal surfaces*, etc. The present volume is designed to explain in their most general form the properties of such systems, with sufficient applications to the "equations of motion," the properties of heat, elasticity, etc., to make their use familiar to the student. No one who wishes to keep himself informed on all valuable advances in recent science should be inattentive to these investigations of LAMÉ. — B.

Tracts, Mathematical and Physical. By HENRY LORD BROUGHAM, LL. D., F. R. S., Member of the National Institute of France, Royal Academy of Naples, Chancellor of the University of Edinburgh. London and Glasgow: Richard Griffin & Co. 1860. 8vo. pp. 304.

The distinguished author's dedication, "To the University of Edinburgh, these tracts, begun while its Pupil, finished when its Head, are inscribed by the Author, in grateful remembrance of benefits conferred of old, and honors of late bestowed," suggest the long period (62 years) within which these tracts were written. Their titles will give the best general idea of their character: General Theorems, chiefly Porisms in the Higher Geometry; KEPLER'S Problem; Dynamical Principle, — Calculus of Partial Differences, — Problem of Three Bodies; Greek Geometry, Ancient Analysis, Porisms; Paradoxes imputed to the Integral Calculus; Architecture of Cells of Bees; Experiments and Investigations on Light and Colors; Inquiries, Analytical and Experimental, on Light; On Forces of Attraction to Several Centres; Meteoric Stones; Central Forces, and Law of the Universe Analytically Investigated; Attraction of Bodies, or Spherical and Non-spherical Surfaces Analytically treated; SIR ISAAC NEWTON, — Grantham Address; Notes. These Essays possess a peculiar value for the student of mathematics and physics; and even those profoundly versed in the subjects of which they treat will not find them devoid of interest.

Editorial Items.

THE following ladies and gentlemen have sent us solutions of the Prize problems in the February number of the Monthly: —

HARRIET S. HAZELTINE, Worcester, Mass., Probs. I., II.
AMANDA M. BENNETT, Saline, Washtenaw Co., Mich., Probs. I., II.
T. E. TOWER, Amherst College, Probs. I., II., III., IV., V.
D. G. BINGHAM, Ellicotteville, N. Y., Probs. I., II.
M. K. BOSWORTH, Marietta College, Ohio, Probs. I., II., V.
CARLOS, West Point, N. Y., Prob. I.
JAMES F. ROBERSON, Indiana University, Bloomington, Probs. II., IV.
GEORGE B. HICKS, Cleveland, Ohio, Probs. III., IV., V.
J. KENDRICK UPTON, New London Institute, N. H., Probs. I., II.
DAVID TROWBRIDGE, Perry City, N. Y., Probs. I., II., III., IV., V.
W. H. SPENCER and C. Y. BALDWIN, Madison University Grammar School, each Prob. I.
CHARLES H. ANDREWS, ——, Prob. I.
H. C. COREY, Exeter, N. H., Probs. I., II.
ASHER B. EVANS, Madison University, Hamilton, N. Y., Probs. III., IV., V.
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24 i ü ö o o o union union common reunion 24

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6 t t t t t time t time t triune t triune 6

Imitate the s and pass from its finishing dot with a double curve to the top of it.

10 s sands s sands s sands s sands s sands 10

7 g g g g goods g goods d gongs d gongs 7

16 my book my book our book our book 16

18 T. Tutu. Tumultuous Tumultuous Triumphant. 18

19 U. U U. Y. Y. Unum Yours Your Yours Unum Yours 19

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	4th Principle – width ⅓ its height.	4th Principle increased in slope forms	left half of a.d.g.q	Slope all down strokes alike.	Except left of o in a.d.g.q	
6	i u ooooooo	o u oooooo	a d g q	and and	gain qua	6
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